Physics 100 Name: ____________________

Mechanical Waves and "western" Music – Part 2: Adding fret marks to a finger board

Formulas/Info:

- To raise frequency by 1 half step, multiply starting frequency by $2^{1/12}$ (or 1.0595)
- To raise frequency by 2 half steps, multiply starting frequency by $2^{2/12}$ (or 1.0595^2)
- To raise frequency by n half steps, multiply starting frequency by $2^{n/12}$ (or 1.0595^n)
- $v = f\lambda$
- $\lambda = 2L_{string}$ The wavelength of the fundamental standing wave in a vibrating string is 2 times the length.
- $L_{string} = \frac{\lambda}{2}$
- The "open string" means the string vibrating over its entire length. When you press the string to the fingerboard, you shorten the vibrating part, so you are no longer playing the open string.

When a string is strummed (or plucked – or bowed), many types of waves travel along it, producing a variety of standing waves. The dominant (loudest) standing wave is called the fundamental. There are also other harmonics (a.k.a. overtones), which have higher pitch.

a. Fundamental (1st Harmonic)

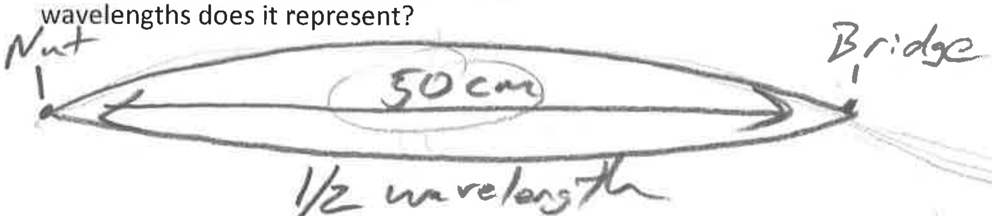
b. 2nd Harmonic

c. 3rd Harmonic

d. 4th Harmonic

36. Suppose an instrument string is **50cm** long, and when the open string is plucked, its frequency is **400hz.**

a. Draw and label a picture of the **fundamental** standing wave that is produced in the vibrating string. Label the nut on the left and the bridge on the right. How many wavelengths does it represent?



b. What is the full wavelength of the waves that are traveling down your string?

$\lambda = 2(50cm) = 100\,cm =$ [illegible] $1m$

c. What is the relationship between string length and the wavelength of the string's fundamental standing wave?

$\lambda = 2L_{string}$ $L_{string} = \frac{\lambda}{2}$

d. What is the speed of those waves?

$v = f\lambda = (400Hz)(1m) = 400\,m/s$

e. The first fret mark on your finger board needs to correspond to a note that is one half-step higher than the open string. What is the frequency of a note one half step higher than the 400hz open string?

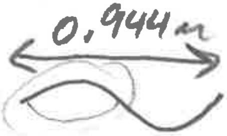
$f_{\frac{1}{2}\,step\,higher} = 2^{1/12}(400Hz) = 1.0595(400Hz)$
$= 423.8Hz$

423.8Hz

$V = \lambda f$

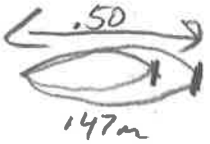
f. In order to play that note, what wavelength must your string have? [hint: you know the string's wave speed]

$\lambda = \frac{V}{f} = \frac{400 m/s}{423.8 Hz} = 0.944m =$



g. How long must the vibrating portion of the string be in order to produce that wavelength?

$L_{string} = \frac{\lambda}{2} = \frac{0.944m}{2} = 0.47m$



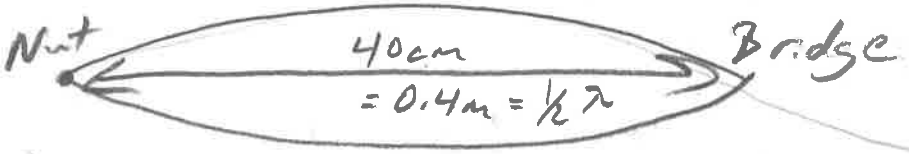
h. How far from the nut should the first fret be located? In other words, by what distance must you shorten your string in order to raise your instrument's pitch by one half step?

$0.5m - 0.47m = 0.03$

Original length New length distance of fret from nut.

37. Suppose an instrument string is **40cm** long, and when the open string is plucked, its frequency is **300hz.**

a. Draw and label a picture of the **fundamental** standing wave that is produced in the vibrating string. Label the nut on the left and the bridge on the right. How many wavelengths does it represent?



b. What is the full wavelength of the waves that are traveling down your string?

$\lambda = 2L_{string} = 2(0.4m) = 0.8m$

c. What is the relationship between string length and the wavelength of the string's fundamental standing wave?

$L_{string} = \frac{\lambda}{2}$ $\lambda = (L_{string})2$

d. What is the speed of those waves?

$v = f\lambda = 300Hz(0.8m) = 240m/s$

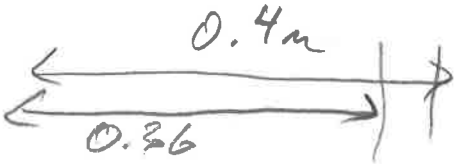
e. The **second** fret mark on your finger board needs to correspond to a note that is two half-step higher than the open string. What is the frequency of a note two half steps higher than the open string?

2 Half Steps ⇒ Multiply Starting Freq by $2^{2/12}$

$300Hz(1.12) = 337 Hz$

f. In order to play that note, what wavelength must your string have? [hint: you know the string's wave speed]

$\lambda = \frac{V}{f} = \frac{240m/s}{337Hz} = 0.71m$



g. How long must the vibrating portion of the string be in order to produce that wavelength?

$$L_{string} = \frac{\lambda}{2} = \frac{0.71m}{2} = \boxed{0.36m}$$

h. How far from the nut should the second fret be located? In other words, by what distance must you shorten your string in order to raise your instrument's pitch by one half step?

$$\underset{\text{Open String}}{\boxed{0.4m}} - \underset{\text{New length}}{0.36m} = 0.04m \leftarrow \text{distance to nut}$$

38. Suppose an instrument string is **60cm** long, and when the open string is plucked, its frequency is **500hz.**

a. What is the full wavelength of the waves that are traveling down your string?

$$\lambda = 2L_{string} = 2(0.6m) = \boxed{1.2m}$$

b. What is the speed of those waves?

$$v = f\lambda = 500Hz(1.2m) = \boxed{600m/s}$$

c. The first fret mark on your finger board needs to correspond to a note that is one half-step higher than the open string. What is the frequency of a note one half step higher than the 500hz open string?

$$500Hz(2^{1/12}) = 500Hz(1.0595) = \boxed{530Hz}$$

d. In order to play that note, what wavelength must your string have? [hint: you know the string's wave speed]

$$\lambda = \frac{v}{f} = \frac{600m/s}{530Hz} = 1.13m$$

e. How long must the vibrating portion of the string be in order to produce that wavelength?

$$L_{string} = \frac{\lambda}{2} = \frac{1.13m}{2} = 0.57m$$

f. How far from the nut should the first fret be located? In other words, by what distance must you shorten your string in order to raise your instrument's pitch by one half step?

$$0.6m - 0.57m = \boxed{0.03m}$$

i. The **second** fret mark on your finger board needs to correspond to a note that is two half-steps higher than the open string. What is the frequency of a note two half steps higher than the open string?

$$500Hz(2^{2/12}) = 500Hz(1.0595^2) = \boxed{561Hz}$$

j. In order to play that note, what wavelength must your string have? [hint: you know the string's wave speed]

$$\lambda = \frac{V}{f} = \frac{600\,m/s}{561\,Hz} = 1.07\,m$$

k. How long must the vibrating portion of the string be in order to produce that wavelength?

$$L_{strng} = \frac{\lambda}{2} = \frac{1.07\,m}{2} = 0.53\,m$$

l. How far from the nut should the second fret be located? In other words, by what distance must you shorten your string in order to raise your instrument's pitch by one half step?

$$0.6\,m - 0.53\,m = \boxed{0.07\,m}$$